International Journal of Theoretical Physics, Vol. 36, No. 2, 1997

Constant-Cutoff Approach to Electric Seagull Terms in the Soliton Hamiltonian

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Received August 12, 1996

We suggest a quantum stabilization method for the $SU(2)$ σ-model, based on the constant-cutoff limit of the cutoff quantization method developed by Balakrishna *et al.*, which avoids the difficulties with the usual soliton boundary conditions pointed out by Iwasaki and Ohyama. We investigate the baryon number $B = 1$ sector of the model and show that after the collective coordinate quantization it admits a stable soliton solution which depends on a single dimensional arbitrary constant. Using the constant-cutoff approach, we then study the $SU(2)$ soliton Hamiltonian, which does not contain the electric seagull terms, and show that if the fields are restricted to the collective subspace, the electric seagull terms are induced in the effective Hamiltonian similarly to the case of the complete Skyrme model. These terms are consistent with gauge invariance and leading-term predictions of the chiral perturbation calculation of the electric polarizability.

1. INTRODUCTION

It was shown by Skyrme (1961, 1962) that baryons can be treated as solitons of a nonlinear chiral theory. The original Lagrangian of the chiral $SU(2)$ σ-model is

$$\mathcal{L} = \frac{F_\pi^2}{16} \operatorname{Tr} \partial_\mu U \, \partial^\mu U^+ \tag{1.1}$$

where

$$U = (2/F_\pi)(\sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}) \tag{1.2}$$

is a unitary operator ($UU^+ = 1$) and F_π is the pion-decay constant. In (1.2), $\sigma = \sigma(\mathbf{r})$ is a scalar meson field and $\boldsymbol{\pi} = \boldsymbol{\pi}(\mathbf{r})$ is the pion isotriplet.

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0020-7748/97/0200-0487$12.50/0

The classical stability of the soliton solution to the chiral σ-model Lagrangian requires an additional ad hoc term, proposed by Skyrme (1961, 1962), to be added to (1.1):

$$\mathcal{L}_{\mathrm{Sk}} = \frac{1}{32e^2} \mathrm{Tr}[U^+\partial_\mu U, U^+\partial_\nu U]^2 \tag{1.3}$$

with a dimensionless parameter e and where $[A, B] = AB - BA$. It was shown by several authors [e.g., Adkins *et al.* (1983); for extensive lists of other references see Holzwarth and Schwesinger (1986) and Nyman and Riska (1990)] that, after collective quantization using the spherically symmetric ansatz

$$U_0(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\mathbf{r}_0 F(r)], \qquad \mathbf{r}_0 = \mathbf{r}/r \tag{1.4}$$

the chiral model, with both (1.1) and (1.3) included, gives good agreement with experiment for several important physical quantities. Thus it should be possible to derive the effective chiral Lagrangian obtained as a sum of (1.1) and (1.3) from a more fundamental theory like QCD. On the other hand, it is not easy to generate a term like (1.3) and give a clear physical meaning to the dimensionless constant e in (1.3) using QCD.

Mignaco and Wulck (1989) (MW) indicated therefore the possibility to build a stable single-baryon ($n = 1$) quantum state in the simple chiral theory with the Skyrme stabilizing term (1.3) omitted. They showed that the chiral angle $F(r)$ is in fact a function of a dimensionless variable $s = \frac{1}{2}\chi''(0)r$, where $\chi''(0)$ is an arbitrary dimensional parameter intimately connected to the usual stability argument against the soliton solution for the nonlinear σ-model Lagrangian.

Using the adiabatically rotated ansatz $U(\mathbf{r}, t) = A(t)U_0(\mathbf{r})A^+(t)$, where $U_0(\mathbf{r})$ is given by (1.4), MW obtained the total energy of the nonlinear σ-model soliton in the form

$$E = \frac{\pi}{4} F_\pi^2 \frac{1}{\chi''(0)} a + \frac{1}{2} \frac{[\chi''(0)]^3}{(\pi/4)F_\pi^2 b} J(J + 1) \tag{1.5}$$

where

$$a = \int_0^\infty \left[\frac{1}{4} s^2 \left(\frac{d\mathcal{F}}{ds}\right)^2 + 8 \sin^2\left(\frac{1}{4} \mathcal{F}\right)\right] dr \tag{1.6}$$

$$b = \int_0^\infty ds \frac{64}{3} s^2 \sin^2\left(\frac{1}{4} \mathcal{F}\right) \tag{1.7}$$

and $\mathcal{F}(s)$ is defined by

$$F(r) = F(s) = -n\pi + \tfrac{1}{4}\mathcal{F}(s) \tag{1.8}$$

The stable minimum of the function (1.5) with respect to the arbitrary dimensional scale parameter $\chi''(0)$ is

$$E = \frac{4}{3} F_\pi \left[\frac{3}{2} \left(\frac{\pi}{4} \right)^2 \frac{a^3}{b} J(J + 1) \right]^{1/4} \tag{1.9}$$

Despite the nonexistence of the stable classical soliton solution to the nonlinear σ-model, it is possible, after collective coordinate quantization, to build a stable chiral soliton at the quantum level, provided that there is a solution $F = F(r)$ which satisfies the soliton boundary conditions, i.e., $F(0) = -n\pi$, $F(\infty) = 0$, such that the integrals (1.6) and (1.7) exist.

However, as pointed out by Iwasaki and Ohyama (1989), the quantum stabilization method in the form proposed by MW is not correct, since in the simple σ-model the conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. In other words, if the condition $F(0) = -\pi$ is satisfied, Iwasaki and Ohyama obtained numerically $F(\infty) \to -\pi/2$, and the chiral phase $F = F(r)$ with correct boundary conditions does not exist.

Iwasaki and Ohyama also proved analytically that both boundary conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. Introducing a new variable $y = 1/r$ into the differential equation for the chiral angle $F = F(r)$, we obtain

$$\frac{d^2F}{dy^2} = \frac{1}{y^2} \sin 2F \tag{1.10}$$

There are two kinds of asymptotic solutions to equation (1.10) around the point $y = 0$, which is called a regular singular point if $\sin 2F \approx 2F$. These solutions are

$$F(y) = \frac{m\pi}{2} + cy^2, \qquad m = \text{even integer} \tag{1.11}$$

$$F(y) = \frac{m\pi}{2} + \sqrt{cy} \cos\left[\frac{\sqrt{7}}{2} \ln(cy) + \alpha \right], \qquad m = \text{odd integer} \tag{1.12}$$

where c is an arbitrary constant and α is a constant to be chosen appropriately. When $F(0) = -n\pi$, then we want to know which of these two solutions is approached by $F(y)$ when $y \to 0$ ($r \to \infty$). In order to answer that question we multiply (1.10) by $y^2F'(y)$, integrate with respect to y from y to ∞, and use $F(0) = -n\pi$. Thus we get

$$y^2F'(y) + \int_y^\infty 2y[F'(y)]^2 \, dy = 1 - \cos[2F(y)] \tag{1.13}$$

Since the left-hand side of (1.13) is always positive, the value of $F(y)$ is always limited to the interval $n\pi - \pi < F(y) < n\pi + \pi$. Taking the limit $y \to 0$, we find that (1.13) is reduced to

$$\int_0^\infty 2y[F'(y)]^2\,dy = 1 - (-1)^m \tag{1.14}$$

where we used (1.11)–(1.12). Since the left-hand side of (1.14) is strictly positive, we must choose an odd integer m. Thus the solution satisfying $F(0) = -n\pi$ approaches (1.12) and we have $F(\infty) \neq 0$. The behavior of the solution (1.11) in the asymptotic region $y \to \infty$ ($r \to 0$) is investigated by multiplying (1.10) by $F'(y)$, integrating from 0 to y, and using (1.11). The result is

$$[F'(y)]^2 = \frac{2\sin^2 F(y)}{y^2} + \int_0^y \frac{2\sin^2 F(y)}{y^3}\,dy \tag{1.15}$$

From (1.15) we see that $F'(y) \to$ const as $y \to \infty$, which means that $F(r) \simeq 1/r$ for $r \to 0$. This solution has a singularity at the origin and cannot satisfy the usual boundary condition $F(0) = -n\pi$.

In Dalarsson (1991a,b, 1992), I suggested a method to resolve this difficulty by introducing a radial modification phase $\varphi = \varphi(r)$ in the ansatz (1.4) as follows:

$$U(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\mathbf{r}_0 F(r) + i\varphi(r)], \qquad \mathbf{r}_0 = \mathbf{r}/r \tag{1.16}$$

Such a method provides a stable chiral quantum soliton, but the resulting model is an entirely noncovariant chiral model, different from the original chiral σ-model.

In the present paper we use the constant-cutoff limit of the cutoff quantization method developed by Balakrishna *et al.* (1991; see also Jain *et al.* (1989) to construct a stable chiral quantum soliton within the original chiral σ-model. Then we apply this method to study the $SU(2)$ soliton Hamiltonian, which does not contain the electric seagull terms, and show that if the fields are restricted to the collective subspace, the electric seagull terms are induced in the effective Hamiltonian, similar to the case of the complete Skyrme model. These terms are consistent with gauge invariance and leading-term predictions of the chiral perturbation calculation of the electric polarizability.

The reason why the cutoff approach to the problem of the chiral quantum soliton works is connected to the fact that the solution $F = F(r)$ which satisfies the boundary condition $F(\infty) = 0$ is singular at $r = 0$. From the physical point of view the chiral quantum model is not applicable to the region about the origin, since in that region there is a quark-dominated bag of the soliton.

However, as argued in Balakrishna *et al.* (1991), when a cutoff ϵ is introduced, then the boundary conditions $F(\epsilon) = -n\pi$ and $F(\infty) = 0$ can be satisfied. They also discussed an interesting analogy with the damped pendulum, showing clearly that as long as $\epsilon > 0$, there is a chiral phase $F = F(r)$ satisfying the above boundary conditions. The asymptotic forms of such a solution are given by Eq. (2.2) in Balakrishna *et al.* (1991). From these asymptotic solutions we immediately see that for $\epsilon \to 0$ the chiral phase diverges at the lower limit.

Different applications of the constant-cutoff approach are discussed in Dalarsson (1993, 1995b–d; 1996a–c).

2. CONSTANT-CUTOFF STABILIZATION

The chiral soliton with baryon number $n = 1$ is given by (1.4), where $F = F(r)$ is the radial chiral phase function satisfying the boundary conditions $F(0) = -\pi$ and $F(\infty) = 0$.

Substituting (1.4) into (1.1), we obtain the static energy of the chiral baryon

$$E_0 = \frac{\pi}{2} F_\pi^2 \int_{\epsilon(t)}^{\infty} dr \left[r^2 \left(\frac{dF}{dr} \right)^2 + 2 \sin^2 F \right] \tag{2.1}$$

In (2.1) we avoid the singularity of the profile function $F = F(r)$ at the origin by introducing the cutoff $\epsilon(t)$ at the lower boundary of the space interval $r \in [0, \infty]$, i.e., by working with the interval $r \in [\epsilon, \infty]$. The cutoff itself is introduced following Balakrishna *et al.* (1991) as a dynamic time-dependent variable.

From (2.1) we obtain the following differential equation for the profile function $F = F(r)$:

$$\frac{d}{dr}\left(r^2 \frac{dF}{dr} \right) = \sin 2F \tag{2.2}$$

with the boundary conditions $F(\epsilon) = -\pi$ and $F(\infty) = 0$, such that the correct soliton number is obtained. The profile function $F = F[r; \epsilon(t)]$ now depends implicitly on time t through $\epsilon(t)$. Thus in the nonlinear σ-model Lagrangian

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_\mu U \, \partial^\mu U^+) \, d^3x \tag{2.3}$$

we use the ansätze

$$U(\mathbf{r}, t) = A(t) U_0(\mathbf{r}, t) A^+(t)$$
$$U^+(\mathbf{r}, t) = A(t) U_0^+(\mathbf{r}, t) A^+(t) \tag{2.4}$$

where

$$U_0(\mathbf{r}, t) = \exp\{i\boldsymbol{\tau}\cdot\mathbf{r}_0 F[r; \epsilon(t)]\} \tag{2.5}$$

The static part of the Lagrangian (2.3), i.e.,

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\nabla U \cdot \nabla U^+)\, d^3x = -E_0 \tag{2.6}$$

is equal to minus the energy E_0 given by (2.1). The kinetic part of the Lagrangian is obtained using (2.4) with (2.5) and it is equal to

$$\begin{aligned} L &= \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_0 U\, \partial_0 U^+)\, d^3x \\ &= bx^2\, \mathrm{Tr}[\partial_0 A\, \partial_0 A^+] + c[\dot{x}(t)]^2 \end{aligned} \tag{2.7}$$

where

$$b = \frac{2\pi}{3} F_\pi^2 \int_1^\infty \sin^2 F\, y^2\, dy, \qquad c = \frac{2\pi}{9} F_\pi^2 \int_1^\infty y^2 \left(\frac{dF}{dy}\right)^2 y^2\, dy \tag{2.8}$$

with $x(t) = [\epsilon(t)]^{3/2}$ and $y = r/\epsilon$. On the other hand, the static energy functional (2.1) can be rewritten as

$$E_0 = ax^{2/3}, \qquad a = \frac{\pi}{2} F_\pi^2 \int_1^\infty \left[y^2\left(\frac{dF}{dy}\right)^2 + 2\sin^2 F\right] dy \tag{2.9}$$

Thus the total Lagrangian of the rotating soliton is given by

$$L = c\dot{x}^2 - ax^{2/3} + 2bx^2\dot{\alpha}_\nu\dot{\alpha}^\nu \tag{2.10}$$

where $\mathrm{Tr}(\partial_0 A\ \partial_0 A^+) = 2\dot{\alpha}_\nu\dot{\alpha}^\nu$ and α_ν ($\nu = 0, 1, 2, 3$) are the collective coordinates defined as in Bhaduri (1988). In the limit of a time-independent cutoff ($\dot{x} \to 0$) we can write

$$H = \frac{\partial L}{\partial \dot{\alpha}^\nu}\dot{\alpha}^\nu - L = ax^{2/3} + 2bx^2\dot{\alpha}_\nu\dot{\alpha}^\nu = ax^{2/3} + \frac{1}{2bx^2} J(J+1) \tag{2.11}$$

where $\langle \mathbf{J}^2 \rangle = J(J+1)$ is the eigenvalue of the square of the soliton laboratory angular momentum. A minimum of (2.11) with respect to the parameter x is reached at

$$x = \left[\frac{2}{3}\frac{ab}{J(J+1)}\right]^{-3/8} \Rightarrow \epsilon^{-1} = \left[\frac{2}{3}\frac{ab}{J(J+1)}\right]^{1/4} \tag{2.12}$$

The energy obtained by substituting (2.12) into (2.11) is given by

$$E = \frac{4}{3}\left[\frac{3}{2}\frac{a^3}{b}J(J + 1)\right]^{1/4} \tag{2.13}$$

This result is identical to the result obtained by Mignaco and Wulck, which is easily seen if we rescale the integrals a and b in such a way that $a \to \frac{1}{4}\pi F_\pi^2 a$, $b \to \frac{1}{2}\pi F_\pi^2 b$, and introduce $f_\pi = 2^{-3/2}F_\pi$. However, in the present approach, as shown in Balakrishna *et al.* (1991), there is a profile function $F = F(y)$ with proper soliton boundary conditions $F(1) = -\pi$ and $F(\infty) = 0$ and the integrals a, b, and c in (2.9)–(2.10) exist and are shown in Balakrishna *et al.* (1991) to be $a = 0.78$ GeV^2, $b = 0.91$ GeV^2, and $c = 1.46$ GeV^2 for $F_\pi = 186$ MeV.

Using (2.13), we obtain the same prediction for the mass ratio of the lowest states as Mignaco and Wulck (1989), which agrees rather well with the empirical mass ratio for the Δ resonance and the nucleon. Furthermore, using the calculated values for the integrals a and b, we obtain the nucleon mass $M(N) = 1167$ MeV, which is about 25% higher than the empirical value of 939 MeV. However, if we choose the pion decay constant equal to $F_\pi = 150$ MeV, we obtain $a = 0.507$ GeV^2 and $b = 0.592$ GeV^2, giving exact agreement with the empirical nucleon mass.

Finally, it is of interest to know how large the constant cutoffs are for the above values of the pion-decay constant in order to check if they are in the physically acceptable ballpark. Using (2.12), it is easily shown that for the nucleons ($J = 1/2$) the cutoffs are equal to

$$\epsilon = \begin{cases} 0.22 \text{ fm} & \text{for} \quad F_\pi = 186 \text{ MeV} \\ 0.27 \text{ fm} & \text{for} \quad F_\pi = 150 \text{ MeV} \end{cases} \tag{2.14}$$

From (2.14) we see that the cutoffs are too small to agree with the size of the nucleon (0.72 fm), as we should expect, since the cutoffs rather indicate the size of the quark-dominated bag in the center of the nucleon. Thus we find that the cutoffs are of reasonable physical size. Since the cutoff is proportional to F_π^{-1}, we see that the pion-decay constant must be less than 57 MeV in order to obtain a cutoff which exceeds the size of the nucleon. Such values of pion-decay constant are not relevant to any physical phenomena.

3. ELECTRIC SEAGULLS IN THE CONSTANT-CUTOFF MODEL

3.1. Introduction

The calculation of the static electromagnetic polarizabilities in the complete Skyrme (1961, 1962) model (CSM) was first performed by Scherer and

Mulders (1992), who argued that the nucleon electromagnetic polarizabilities provide important information about the nucleon structure. In the recent years there has been significant improvement in the experimental measurements of these quantities. The quality of the predictions for static electromagnetic polarizabilities is a significant test of any model for the description of the nucleon.

The constant-cutoff approach to the calculation of static electromagnetic polarizabilities was presented in Dalarsson (1995a). In both the CSM Scherer and Mulders (1992) and Dalarsson (1995a) the dominant contribution to the polarizability is a seagull contribution—i.e., a term in the effective Hamiltonian quadratic in the external electric field. The method used was to introduce an external electric field in the z direction by choosing $A_0 = -Ez$, and then to couple it to the Lagrangian density of the model in the usual way. The seagull term in the collective Hamiltonian was then simply identified as minus the seagull contribution from the collective Lagrangian density since this term has no time derivatives.

On the other hand, a recent publication L'vov (1993) argues that the existence of electric seagulls violates the local gauge invariance. Furthermore, L'vov (1993) explicitly demonstrated that the Hamiltonian of the Skyrme model, and of course even the simplified Hamiltonian in the constant-cutoff approach, do not have seagulls.

However, the behavior of the electric polarizability in the chiral limit and for large N_c supports the results obtained in Scherer and Mulders (1992) and Dalarsson (1995a) and shows that the seagulls appear to be necessary to obtain the correct result in the chiral and large-N_c limits.

The purpose of the present paper is to resolve this apparent contradiction in the case of the constant-cutoff approach to the simplified Skyrme model. The contradiction is resolved by observing that the present model is studied in the large-N_c limit, where there is a collective manifold of configurations which determine the low-energy properties, e.g., a space of rotating hedgehogs.

The main point is now that even though the Hamiltonian of the original model has no seagulls, the effective Hamiltonian as a function of the collective variables has seagull terms. The argument put forward in L'vov (1993) is based on the fact that the field theory is local, while the constraint to the collective manifold depends on some spatial integrals and the effective Hamiltonian does not belong to a local field theory.

Furthermore, the apparent difficulty with assuming that the electric seagull term in $\mathcal{H}^{\text{coll}}$ is simply minus the seagull term in $\mathcal{L}^{\text{coll}}$ must be addressed, since the case of a constant A_0 is a clear case where this procedure is not correct. This difficulty is immediately resolved in some specific cases, e.g., when a term in the collective Hamiltonian, linear in the external field, vanishes for all values of collective variables due to a symmetry. It will be shown that this is the case if all configurations in the collective manifold have the same parity and the external field is of odd parity, which applies to the present model in a constant electric field.

3.2. Seagull Terms in the Collective Hamiltonian

The Lagrangian of the simplified Skyrme model with massive pions is obtained from (1.1) by adding the chiral-symmetry-breaking mass term, and is given by

$$\mathcal{L} = \frac{F_\pi^2}{16} \operatorname{Tr} \partial_\mu U \, \partial^\mu U^+ + \frac{m_\pi^2 F_\pi^2}{16} \operatorname{Tr}(U + U^+ - 2) \tag{3.1}$$

When an external electromagnetic field is present then, by minimal substitution (Dalarsson, 1995a) and using (1.2), we obtain the total Hamiltonian density as follows:

$$\mathcal{H} = \frac{1}{2} \Phi_i G_{ij}^{-1} \Phi_j + \mathcal{M}(\pi) - eA_0(\rho + J_i^0 G_{ij}^{-1} \Phi_j) - e^2 A_0^2 (\Gamma^{00} - J_i^0 G_{ij}^{-1} J_j^0) \tag{3.2}$$

where $\Phi_i = G_{ij}\dot{\pi}_j + eA_0 J_i^0$ are the canonical momenta conjugate to the field $\pi_i = \pi_i(\mathbf{r}, t)$, $\rho = -(2\pi r)^{-2} \sin^2 F \, dF/dr$ is the charge density, Γ^{00} is the seagull term defined in Saito and Uehara (1994), and we define

$$J_i^0 = \epsilon_{3jk} \pi_j G_{ki} \tag{3.3}$$

$$\mathcal{M}(\pi) = \tfrac{1}{2} \partial_k \pi_i G_{ij} \partial_k \pi_j + \tfrac{1}{2} m_\pi^2 \pi_j^2 \tag{3.4}$$

with the metric G_{ij} given by

$$G_{ij} = \delta_{ij} + \frac{\pi_i \pi_j}{\sigma^2} = \delta_{ij} + \tan^2 F \, \hat{r}_i \hat{r}_j \tag{3.5}$$

It should be noted that in (3.2) we have taken $A_j = 0$ ($j = 1, 2, 3$), i.e., we have assumed that the external magnetic field is zero. Furthermore, we keep only the leading terms in the $1/N_c$ expansion as in Dalarsson (1995a), i.e., we neglect several higher order terms. For the discussion of these terms see Scherer and Mulders (1992) and Dalarsson (1995a).

It is shown in Saito and Uehara (1994) by explicit calculation that $\Gamma^{00} = -J_i^0 G_{ij}^{-1} J_j^0$ and that the third term in (3.2) vanishes. Thus at the level of the Hamiltonian without constraint to the specific collective manifold there are no electric seagull terms. However, in the present model baryons are described as slowly rotating field configurations

$$\sigma = \tfrac{1}{2} F_\pi \cos F, \qquad \pi_j = \tfrac{1}{2} F_\pi \sin F \, R_{ji} r_0^i \tag{3.6}$$

where R_{ij} is the time-dependent rotation matrix. We will now show that (3.6) as a constraint to the specific manifold in the Hilbert space of the π-fields leads to the appearance of electric seagull terms. Replacing this hedgehog configuration into the Hamiltonian density (3.2), we obtain the total Hamiltonian

$$H^{\text{coll}} = E_s + \frac{1}{2}\Omega\boldsymbol{\omega}^2 + \int d^3\mathbf{r}\, eA_0 \frac{1}{4\pi r^2}\sin^2 F\frac{dF}{dr} - \frac{1}{8}F_\pi^2\int d^3\mathbf{r}\, e^2A_0^2\sin^2F\,[1 - R_{3j}R_{3k}r_0^jr_0^k] \tag{3.7}$$

where the inertia Ω and the static energy E_s of the soliton are given by

$$\Omega = \frac{2\pi}{3}F_\pi^2\int_\epsilon^\infty dr\, r^2\sin^2F \tag{3.8}$$

$$E_s = \frac{\pi}{2}F_\pi^2\int_\epsilon^\infty dr\left[r^2\left(\frac{dF}{dr}\right)^2 + 2\sin^2F + 4m_\pi^2r^2\sin^2\frac{F}{2}\right] \tag{3.9}$$

and $\boldsymbol{\omega}$ is the angular velocity defined by $R_{mi}R_{mj} = \epsilon_{ijk}\omega_k$.

Although the third term in (3.2) was shown in Saito and Uehara (1994) to vanish, in (3.7) there is still a term proportional to e^2, i.e., a seagull term. It has its origin in the first two terms of (3.2).

In order to write down a proper canonical form of the Hamiltonian (3.7), we introduce the momentum conjugate to the angular velocity $\boldsymbol{\omega}$ as the isospin $\mathbf{T}$, given by

$$T_i = \Omega\omega_i + \frac{1}{4}F_\pi^2\int d^3\mathbf{r}\, eA_0F_\pi^2\sin^2F\,(\delta_{ij} - r_{0i}r_{0j})R_{3j} \tag{3.10}$$

Using (3.10), we find that the Hamiltonian (3.7) becomes

$$\begin{aligned} H^{\text{coll}} &= H^{(0)} + H^{(1)} + H^{(2)} \\ &= E_s + \frac{T^2}{2\Omega} \\ &\quad + \int d^3\mathbf{r}\, eA_0\left[\frac{1}{4\pi r^2}\sin^2F\frac{dF}{dr} - \frac{F_\pi^2}{4\Omega}\sin^2F\,(\delta_{ij} - r_{0i}r_{0j})R_{3j}T_i\right] \\ &\quad + \frac{\Omega}{2}\left[\int d^3\mathbf{r}\, eA_0\frac{F_\pi^2}{4\Omega}\sin^2F\,(\delta_{ij} - r_{0i}r_{0j})\right] \\ &\quad \times\left[\int d^3\mathbf{r}\, eA_0\frac{F_\pi^2}{4\Omega}\sin^2F\,(\delta_{ik} - r_{0i}r_{0k})\right]R_{3j}R_{3k} \\ &\quad - \frac{1}{8}F_\pi^2\int d^3\mathbf{r}\, e^2A_0^2\sin^2F\,(1 - R_{3j}R_{3k}r_0^jr_0^k) \end{aligned} \tag{3.11}$$

where $H^{(i)}$ ($i = 0, 1, 2$) are terms in the Hamiltonian proportional to e^i ($i = 0, 1, 2$), which are easily identified in (3.11).

Now we can turn to the two particular cases of interest. The first case is discussed in L'vov (1993), where A_0 = const is a constant field. In that case we obtain

$$H^{(1)}(A_0{=}\text{const}) = -eA_0(\tfrac{1}{2} + T_3), \qquad H^{(2)}(A_0{=}\text{const}) = 0 \tag{3.12}$$

where we used $T_3 = R_{3j}T_j$. Thus in this case the seagull term does vanish and the remaining Hamiltonian describes the interaction of the constant field A_0 with the baryon electric charge.

The second case, which is of more interest, is that of a constant electric field $\mathscr{E} = \mathscr{E}\mathbf{z}_0$ with $A_\mu = (-z\mathscr{E}, 0, 0, 0)$ used in Scherer and Mulders (1992) and Dalarsson (1995a). Using here the relation

$$R_{33}^2 = \tfrac{1}{3} + \tfrac{2}{3}D_{0,0}^{(2)}(\alpha, \beta, \gamma) \tag{3.13}$$

where $D_{0,0}^{(2)}$ is the corresponding D-matrix defined as in Dalarsson (1995a) and (α, β, γ) are Euler angles, we obtain

$$H^{(1)}(A_0 = -z\mathscr{E}) = 0, \qquad H^{(2)}(A_0 = -z\mathscr{E}) = -\tfrac{1}{2}g^e\mathscr{E}^2[1 - \tfrac{2}{5}D_{0,0}^{(2)}] \tag{3.14}$$

where

$$g^e = \frac{e^2}{18}F_\pi^2 \int d^3\mathbf{r}\, r^2 \sin^2 F \tag{3.15}$$

and a more detailed account of the calculation techniques employed to find the results (3.14) and (3.15) can be found in Dalarsson (1995a).

The results (3.14) coincide with those obtained in Eq. (5.11) in Dalarsson (1995a) and give the same results for the electric polarizability of nucleons $\alpha = g^e$, equal to the parameter g^e given in (3.15), since for nucleons the matrix element of $D_{0,0}^{(2)}$ vanishes, and for the electric polarizabilities of Δ-particles

$$\alpha = \begin{cases} g^e(1 - 2/25), & |T_3| = |J_3| \\ g^e(1 + 2/25), & |T_3| \neq |J_3| \end{cases} \tag{3.16}$$

The numerical results for electric polarizabilities of nucleons and Δ-particles obtained in Dalarsson (1995a) thus remain valid despite the objections posed in L'vov (1993) and Saito and Uehara (1994). In the present paper they are confirmed in a more rigorous way.

4. CONCLUSIONS

In conclusion we have demonstrated that, even though a fundamental Hamiltonian of the constant-cutoff approach to the $SU(2)$ σ-model does not contain any electric seagull terms, the constraint to the collective subspace of rotating hedgehogs gives rise to the electric seagull terms in the collective Hamiltonian.

Furthermore, we have shown that such terms are in full agreement with the corresponding terms obtained using the somewhat naive procedure in Dalarsson (1995a).

Thus the possibility to use the Skyrme model for the calculation of the electromagnetic polarizabilities of nucleons and Δ-particles, without the use of the Skyrme-stabilizing term proportional to e^{-2}, which makes practical calculations more complicated and generates nonadiabatic corrections to the first-order isovector terms, remains valid despite the objections posed in L'vov (1993) and Saito and Uehara (1994). On the other hand, these objections are in general correct for other cases than the one of the constant electric field in the z-direction, which was explicitly demonstrated in the case of a constant A_0 field, where there are no seagull terms.

Finally, the numerical results obtained in Dalarsson (1995a) are still valid and are now based on a more rigorous analytic treatment than the one used there.

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